1.3 | Unit Conversion

Learning Objectives

By the end of this section, you will be able to:

- Use conversion factors to express the value of a given quantity in different units.

It is often necessary to convert from one unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you may need to convert units of feet or meters to miles.

Let's consider a simple example of how to convert units. Suppose we want to convert 80 m to kilometers. The first thing to do is to list the units you have and the units to which you want to convert. In this case, we have units in *meters* and we want to convert to *kilometers*. Next, we need to determine a conversion factor relating meters to kilometers. A **conversion factor** is a ratio that expresses how many of one unit are equal to another unit. For example, there are 12 in. in 1 ft, 1609 m in 1 mi, 100 cm in 1 m, 60 s in 1 min, and so on. Refer to **Appendix B** for a more complete list of conversion factors. In this case, we know that there are 1000 m in 1 km. Now we can set up our unit conversion. We write the units we have and then multiply them by the conversion factor so the units cancel out, as shown:

$$80\,\cancel{\text{m}} \times \frac{1\text{ km}}{1000\,\cancel{\text{m}}} = 0.080\text{ km}.$$

Note that the unwanted meter unit cancels, leaving only the desired kilometer unit. You can use this method to convert between any type of unit. Now, the conversion of 80 m to kilometers is simply the use of a metric prefix, as we saw in the preceding section, so we can get the same answer just as easily by noting that

$$80\text{ m} = 8.0 \times 10^1\text{ m} = 8.0 \times 10^{-2}\text{ km} = 0.080\text{ km},$$

since "kilo-" means 10^3 (see **Table 1.2**) and $1 = -2 + 3.$ However, using conversion factors is handy when converting between units that are not metric or when converting between derived units, as the following examples illustrate.

Example 1.2

Converting Nonmetric Units to Metric

The distance from the university to home is 10 mi and it usually takes 20 min to drive this distance. Calculate the average speed in meters per second (m/s). (*Note:* Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units, then we can get the average speed into the desired units by picking the correct conversion factors and multiplying by them. The correct conversion factors are those that cancel the unwanted units and leave the desired units in their place. In this case, we want to convert miles to meters, so we need to know the fact that there are 1609 m in 1 mi. We also want to convert minutes to seconds, so we use the conversion of 60 s in 1 min.

Solution

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now. Average speed and other motion concepts are covered in later chapters.) In equation form,

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}}.$$

2. Substitute the given values for distance and time:

$$\text{Average speed} = \frac{10\text{ mi}}{20\text{ min}} = 0.50\,\frac{\text{mi}}{\text{min}}.$$

3. Convert miles per minute to meters per second by multiplying by the conversion factor that cancels miles and leave meters, and also by the conversion factor that cancels minutes and leave seconds:

$$0.50\,\frac{\cancel{\text{mile}}}{\cancel{\text{min}}} \times \frac{1609\text{ m}}{1\,\cancel{\text{mile}}} \times \frac{1\,\cancel{\text{min}}}{60\text{ s}} = \frac{(0.50)(1609)}{60}\text{ m/s} = 13\text{ m/s}.$$

Significance

Check the answer in the following ways:

1. Be sure the units in the unit conversion cancel correctly. If the unit conversion factor was written upside down, the units do not cancel correctly in the equation. We see the "miles" in the numerator in 0.50 mi/min cancels the "mile" in the denominator in the first conversion factor. Also, the "min" in the denominator in 0.50 mi/min cancels the "min" in the numerator in the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of meters per second and, after the cancellations, the only units left are a meter (m) in the numerator and a second (s) in the denominator, so we have indeed obtained these units.

1.2 Check Your Understanding Light travels about 9 Pm in a year. Given that a year is about $3 \times 10^7\,\text{s}$, what is the speed of light in meters per second?

Example 1.3

Converting between Metric Units

The density of iron is 7.86 g/cm^3 under standard conditions. Convert this to kg/m^3.

Strategy

We need to convert grams to kilograms and cubic centimeters to cubic meters. The conversion factors we need are $1\text{ kg} = 10^3\,\text{g}$ and $1\text{ cm} = 10^{-2}\,\text{m}.$ However, we are dealing with cubic centimeters $(\text{cm}^3 = \text{cm} \times \text{cm} \times \text{cm})$, so we have to use the second conversion factor three times (that is, we need to cube it). The idea is still to multiply by the conversion factors in such a way that they cancel the units we want to get rid of and introduce the units we want to keep.

Solution

$$7.86\frac{\cancel{\text{g}}}{\cancel{\text{cm}}^3} \times \frac{\text{kg}}{10^3\,\cancel{\text{g}}} \times \left(\frac{\cancel{\text{cm}}}{10^{-2}\,\text{m}}\right)^3 = \frac{7.86}{(10^3)(10^{-6})}\text{kg/m}^3 = 7.86 \times 10^3\text{ kg/m}^3$$

Significance

Remember, it's always important to check the answer.

1. Be sure to cancel the units in the unit conversion correctly. We see that the gram ("g") in the numerator in 7.86 g/cm^3 cancels the "g" in the denominator in the first conversion factor. Also, the three factors of "cm" in the denominator in 7.86 g/cm^3 cancel with the three factors of "cm" in the numerator that we get by cubing the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked for us to convert to kilograms per cubic meter. After the cancellations just described, we see the only units we have left are "kg" in the numerator and three factors of "m" in the denominator (that is, one factor of "m" cubed, or "m^3"). Therefore, the units on the final answer are correct.

1.3 Check Your Understanding We know from **Figure 1.4** that the diameter of Earth is on the order of 10^7 m, so the order of magnitude of its surface area is 10^{14} m^2. What is that in square kilometers (that is, km^2)? (Try doing this both by converting 10^7 m to km and then squaring it and then by converting 10^{14} m^2 directly to square kilometers. You should get the same answer both ways.)

Unit conversions may not seem very interesting, but not doing them can be costly. One famous example of this situation was seen with the *Mars Climate Orbiter*. This probe was launched by NASA on December 11, 1998. On September 23, 1999, while attempting to guide the probe into its planned orbit around Mars, NASA lost contact with it. Subsequent investigations showed a piece of software called SM_FORCES (or "small forces") was recording thruster performance data in the English units of pound-seconds (lb-s). However, other pieces of software that used these values for course corrections expected them to be recorded in the SI units of newton-seconds (N-s), as dictated in the software interface protocols. This error caused the probe to follow a very different trajectory from what NASA thought it was following, which most likely caused the probe either to burn up in the Martian atmosphere or to shoot out into space. This failure to pay attention to unit conversions cost hundreds of millions of dollars, not to mention all the time invested by the scientists and engineers who worked on the project.

1.4 Check Your Understanding Given that 1 lb (pound) is 4.45 N, were the numbers being output by SM_FORCES too big or too small?

1.4 | Dimensional Analysis

Learning Objectives

By the end of this section, you will be able to:

- Find the dimensions of a mathematical expression involving physical quantities.
- Determine whether an equation involving physical quantities is dimensionally consistent.

The **dimension** of any physical quantity expresses its dependence on the base quantities as a product of symbols (or powers of symbols) representing the base quantities. Table 1.3 lists the base quantities and the symbols used for their dimension. For example, a measurement of length is said to have dimension L or L^1, a measurement of mass has dimension M or M^1, and a measurement of time has dimension T or T^1. Like units, dimensions obey the rules of algebra. Thus, area is the product of two lengths and so has dimension L^2, or length squared. Similarly, volume is the product of three lengths and has dimension L^3, or length cubed. Speed has dimension length over time, L/T or LT^{-1}. Volumetric mass density has dimension M/L^3 or ML^{-3}, or mass over length cubed. In general, the dimension of any physical quantity can be written as $\mathrm{L}^a\mathrm{M}^b\mathrm{T}^c\mathrm{I}^d\Theta^e\mathrm{N}^f\mathrm{J}^g$ for some powers $a, b, c, d, e, f,$ and g. We can write the dimensions of a length in this form with $a = 1$ and the remaining six powers all set equal to zero: $\mathrm{L}^1 = \mathrm{L}^1\mathrm{M}^0\mathrm{T}^0\mathrm{I}^0\Theta^0\mathrm{N}^0\mathrm{J}^0$. Any quantity with a dimension that can be written so that all seven powers are zero (that is, its dimension is $\mathrm{L}^0\mathrm{M}^0\mathrm{T}^0\mathrm{I}^0\Theta^0\mathrm{N}^0\mathrm{J}^0$) is called **dimensionless** (or sometimes "of dimension 1," because anything raised to the zero power is one). Physicists often call dimensionless quantities *pure numbers*.

Base Quantity	Symbol for Dimension
Length	L
Mass	M
Time	T
Current	I
Thermodynamic temperature	Θ
Amount of substance	N
Luminous intensity	J

Table 1.3 Base Quantities and Their Dimensions

Physicists often use square brackets around the symbol for a physical quantity to represent the dimensions of that quantity. For example, if r is the radius of a cylinder and h is its height, then we write $[r] = \mathrm{L}$ and $[h] = \mathrm{L}$ to indicate the dimensions of the radius and height are both those of length, or L. Similarly, if we use the symbol A for the surface area of a cylinder and V for its volume, then $[A] = \mathrm{L}^2$ and $[V] = \mathrm{L}^3$. If we use the symbol m for the mass of the cylinder and ρ